Assessing Horton and Kostiakov models focused on estimating soil water infiltration

Avaliação dos modelos de Horton e de Kostiakov para estimativa da infiltração de água no solo

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ABSTRACT: Water infiltration into the soil plays key role in proper soil and water conservation and management processes. The aim of the current study is to assess water behavior in soil infiltration processes by using the Horton and Kostiakov models. Double-ring infiltrometer method was used to determine water infiltration into the soil. In addition, soil samples were collected at depths of 0-10 cm, 10-20 cm and 20-30 cm, to determine soil moisture. Cumulative infiltration (IA) and instantaneous infiltration rates per time interval (TiR) were calculated. Equations describing the investigated phenomena were adjusted, according to the Horton and Kostiakov models. High water infiltration rate was observed at the beginning of the process, likely due to low moisture level at the topsoil layer. Both models tend to underestimate the initial infiltration rate value. However, the Horton model tends to overestimate most of the instantaneous infiltration values up to 25 minutes. The Kostiakov model, on the other hand, tends to overestimate most of the instantaneous infiltration rate values after 80 minutes. Despite the large fluctuation observed in TiR values in the soil, the assessed models have satisfactorily described the infiltration behavior at the experimental site. Based on the heat map analysis, the Kostiakov model was the one capable of determining the infiltration values closest to actual one.

Keywords: Infiltrometer; Infiltration rate; Cambisol.

RESUMO: A infiltração de água no solo exerce importante papel na conservação e manejo adequados do solo e da água. Nesse sentido, este estudo teve como objetivo avaliar o comportamento da infiltração de água no solo, estimada pelos modelos de Horton e Kostiakov. Para determinação da infiltração, empregou-se o método do infiltrometro de duplo anel. Paralelamente ao ensaio, foram coletadas amostras de solo nas profundidades de 0-10 cm, 10-20 cm e 20-30 cm, para determinação da umidade do solo. Foram calculadas a infiltração acumulada (IA) e as taxas de infiltração instantâneas por intervalo de tempo (TiR), bem como efetuou-se o ajuste das equações que descrevem os fenômenos, conforme os modelos de Horton e Kostiakov. Se observou elevada taxa de infiltração de água no início do processo, que pode ser resultante da baixa umidade na camada superficial do solo. Ambos os modelos tendem a subestimar o valor inicial da taxa de infiltração. No entanto, observa-se que até 25 minutos, o modelo de Horton tende a superestimar a maioria dos valores das infiltrações instantâneas. Já o modelo de Kostiakov tende a superestimar a maior parte dos valores das taxas de infiltração instantâneas a partir de 80 minutos. Apesar da grande oscilação nos valores das TiR no solo, os modelos descreveram de forma satisfatória o comportamento da taxa de infiltração na área experimental e por meio da análise do mapa de calor, entende-se que o modelo de Kostiakov é o que mais se aproxima da infiltração real.

Palavras-chave: Infiltrometro; Taxa de infiltração; Cambissolo.
INTRODUCTION

Water dynamics in the soil is an essential continuous process both for studies focused on investigating surface runoff and flooding, mainly in urbanized areas, and for studies conducted in agricultural areas, since it can affect water and nutrient availability for plants. It can be described, among other methods, based on water infiltration into the soil (NETTO et al., 2000), which, according to Cecílio et al. (2013), refers to the process through which surface water enters the environment.

Several factors can influence this infiltration process, namely: rainfall features (intensity and volume), soil physical properties (texture, structure, compaction degree) and agricultural management method (conventional, direct planting or pasture) (SANTOS; PEREIRA, 2013; BRANDÃO et al., 2006; MANCUSO et al., 2014).

Some soil preparation methods can favor soil compaction in cultivated areas and reduce water infiltration capacity. According to Oliveira, Silva and Mello (2020), low water infiltration capacity can increase surface runoff and hinder soil groundwater recharge processes. Furthermore, it can open room for water-related erosion, since soil cover removal exposes its surface and makes it susceptible to the action of rainfall events (PANACHUKI et al., 2011).

It is essential investigating the water behavior in soil infiltration processes, based on infiltration rates, to help implementing measures aimed at promoting soil conservation, at planning drainage projects and at generating data that can be used to estimate soil real water storage potential (PAIXÃO et al., 2009).

Ring infiltrometer is the method mostly used to determine soil infiltration, among the existing ones, given its easy execution (GONDIM et al., 2010). This method lies on using two cylinders with different diameters, one internal and one external, to reduce the output of water laterally infiltrated by the internal cylinder (FAGUNDES et al., 2012; BAZZO; HORN, 2017). Infiltration readings are carried out, at pre-established times, in the cylinder with the smallest diameter, and it is done to enable assessing water vertically infiltrated in the medium.

The infiltration process can also be portrayed through physical or empirical mathematical models (FELIZARDO et al., 2020), with emphasis on the Horton and Kostiakov models, whose parameters can be calculated through theoretical equations or estimated through regression model, based on pre-determined infiltration data in the field (TOMASINI et al., 2010). Horton's empirical model, which is described in the form of an exponential function, is widely used to predict water behavior in soil infiltration processes (PAIXÃO et al., 2009; MELLEK et al., 2014). The Kostiakov model, in its turn, uses a potential equation to estimate accumulated infiltration or infiltration rate based on time. However, it has limitations, since it cannot be applied to soils other than the one that enabled estimating the equation parameters or its use in soils presenting initial moisture conditions different from these ones, based o which, the assessed parameters were determined (ALMEIDA, 2017).

According to Oliveira, Soares and Holanda (2018), the method and/or model to be adopted can change depending on the region it will be applied in. Thus, it is necessary conducting infiltration behavior studies (in situ and based on different models) to recommend the most accurate model to be applied in the region of interest.

Results of studies focused on investigating infiltration behavior can be translated into infiltration curves, which can be used in hydrological studies, as well as in urban drainage or irrigation design projects (FELIZARDO et al., 2020).
In light of the foregoing, the aim of the current study was to investigate the suitability of the Horton and Kostiakov models to predict water infiltration behavior in Cambisol, in an area close to the crop region in Irati City-PR.

MATERIAL AND METHODS

Study site

The study was carried out in a location close to private areas of agricultural crops, at State University of Center-West, UNICENTRO, Irati campus - PR. The soil in this region is originally classified as Cambisol. It is featured by mineral materials, with incipient B horizon underlying any surface horizon type (EMBRAPA, 2018). The area defined for the current study was fallow and presented a thin layer of organic material mainly composed of tree leaves.

According to the Köppen climate classification, climate in the region is of the Cfb type, with mild summers and winters presenting frequent incidence of severe frost (IRATI, 2020). Mean annual rainfall reaches 1,430 mm and mean annual temperature reaches 18ºC (CLIMATE-DATE, 2019).

Assessing water infiltration into the soil

Double-ring infiltrometer method was used to investigate the behavior of water infiltration into the soil, based on water application by flooding, as described by Bernardo, Soares and Mantovani (2006) and Garcez and Alvarez (1988). Cylinders measuring 10 cm and 30 cm in diameter and 15 cm in height (Figures 1A and 1B) were concentrically sunk into the ground at depth close to 5 cm (Figure 1C). A mini-PVC water reservoir (10 cm in diameter with useful volume of 0.186 m³) was installed in these cylinders (Figure 1D).

Table 1 presents the respective test times and each time interval used to read the infiltrated water depth.

<table>
<thead>
<tr>
<th>Test time (minutes)</th>
<th>Time interval for ruler readings (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 10</td>
<td>1</td>
</tr>
<tr>
<td>10 to 38</td>
<td>2</td>
</tr>
<tr>
<td>38 to 59</td>
<td>3</td>
</tr>
<tr>
<td>59 to 163</td>
<td>4</td>
</tr>
<tr>
<td>163 to 193</td>
<td>5</td>
</tr>
</tbody>
</table>
Figure 1. Set of concentric rings (A and B); cylinders’ installation in the experimental area (C); set of rings installed in the ground with the mini reservoir (D).

Accumulated infiltration and real infiltration rate

Accumulated infiltration (AI) and instantaneous infiltration rates (TiR) per time interval were calculated based on data collected in the field test. Equations describing the investigated phenomenon were adjusted based on models proposed by Horton and Kostiakov (BERNARDO; SOARES; MANTOVANI, 2006). Real vertical soil infiltration rate (TiR) was calculated through the following expression (Equation 1):

\[ TiR = \frac{\text{Infiltrated blade}}{\text{time interval}} \]  

(1)

Wherein: TiR is the rate of water infiltration into the soil (in mm h\(^{-1}\)); Infiltrated depth is the amount of water infiltrated over a period–of-time (in mm); and time interval represents the time interval set for soil water infiltration readings (in hours).

Infiltration rates were estimated by taking into consideration the determination of real infiltration rates, based on using Horton’s equation expressed through equation 2:

\[ f = f_c + (f_0 - f_c).e^{-k.t} \]  

(2)

Wherein: \( f \) is the infiltration rate at any instant (in mm h\(^{-1}\)); \( f_c \) is the final infiltration rate (in mm h\(^{-1}\)); \( f_0 \) is the initial infiltration rate (in mm h\(^{-1}\)); \( k \) is the infiltration decay constant and \( t \) is the infiltration time taken into consideration (in hours).

Accumulated infiltration \( I \) was described through the Potential Equation (or Kostiakov – 1932) expressed through equation 3:

\[ I = k.t^a \]  

(3)
Wherein: I is the accumulated infiltration (in cm); k is the constant depending on the soil; t is the infiltration time (in minutes) and a is the constant depending on the soil - it ranges from 0 to 1.

By deriving the Potential Equation accounting for describing the accumulated infiltration, it was possible finding the infiltration rate estimated by the Kostiakov model, through equation 4:

$$\text{TiE} = a. k. t^{a-1}$$  \tag{4}

### Initial soil moisture

Previous studies have shown soil moisture interference with both its initial and accumulated infiltration rates (PANACHUKI et al., 2006; ALMEIDA et al., 2020). Soil samples were collected at depths of 0-10 cm, 10-20 cm and 20-30 cm, close to the installed cylinders, at the same day the infiltration tests were carried out. It was done to determine its current moisture level.

The gravimetric method (MOREIRA; TEIXEIRA; DONAGEMMA, 2017) described by Equation 5 was used to determine soil moisture level before infiltration tests were carried out in the experimental area:

$$U(\text{kg kg}^{-1}) = \frac{M_w}{M_s} \quad \text{or} \quad U(\%) = \frac{M_w}{M_s} \times 100$$  \tag{5}

Wherein: U corresponds to the moisture content in the sample (in kg kg\(^{-1}\) or %); Ms is the weight of the sample dried at 105 °C until it reached constant weight (in g); and Ma refers to water weight in the sample (in g) - which is given by the weight difference between the wet (under the conditions it was collected in the field) and the dry sample.

### Results analysis

Regression method was used to assess the behavior of water infiltration accumulated in the soil overtime. Water infiltration estimated through the adopted models was assessed by comparing the curves of real infiltration rates to estimated infiltration rates, based on time, plotted in a graph.

Horton model parameters were estimated by minimizing the sum of the square of differences between real values and infiltration values estimated by the model, by using the Excel solver tool. Kostiakov model coefficients “a” and “k” were determined based on the linear regression method described by Bernardo, Soares and Mantovani (2006), by taking into consideration values determined in the field.

The Horton and Kostiakov models were compared based on using coefficient of determination ($R^2$), mean squared error (MSE) and mean absolute error (EMA).

All data were subjected to hierarchical cluster analysis, based on using the K-means algorithm, to define the closest model to the real behavior.

This technique was selected because it does not require Gaussianity, homogeneity of variances or sample independence assumptions.

Along with the hierarchical grouping, a heat map was plotted to describes the values recorded in the research, through gradual color variations. Higher values were represented by dark colors, whereas lower values were represented by light colors. Analyses were carried out in RStudio software (R Core Team, 2023).
RESULTS AND DISCUSSION

Figure 2 presents results recorded for accumulated infiltration (AI) and real rate of instantaneous water infiltration into the soil based on time.

Figure 2. Accumulated infiltration and real rate of water infiltration into the soil based on time.

As expected, there was inverse behavior between infiltration rate and infiltration of water accumulated in the soil - total water accumulation reached 1.041 mm over 3-hour testing (193 minutes). AI values close to the ones observed in the current study were reported by Gondim et al. (2010), who investigated water infiltration based on the ring infiltrometer method. They observed total accumulated infiltration equal to 1.184 mm over a longer test time (7 hours and 25 minutes).

The AI representative curve presented good fit, with coefficient of determination equal to 99%. It was represented by equation: $AI = 401.58.T^{0.8451}$ ($T$ was expressed in hours), and it has evidenced that the adjusted equation can accurately predict the behavior of water infiltration into the soil.

Similar results were observed by Silva et al. (2019), who investigated soil infiltration based on the Kostiakov model. They recorded $R^2$ higher than 95% to adjust AI equations to different management systems (forest, forest-crop, crop rotation and pasture). Nunes et al. (2012) have also identified good AI equation fit. They observed $R^2$ equal to 99% for both bare soil and pasture, and it strengthened the perception that this model is a good predictor of the behavior of water infiltration accumulated in the soil.

Almeida et al. (2020) investigated a Cambisol in the semi-arid region and found AI equal to 20.2 cm and 53.09 cm in wet and dry soil, respectively, over 2-hour testing. This finding suggested that the previous moisture level in the soil interfered with the final AI value.

The drier the soil, the higher its infiltration capacity. Based on Figure 3, the soil was drier at the top layer (0-10 cm), and it may explain the high values observed for both...
the initial infiltration rate (840.0 mm.h^{-1}) and for the final accumulated infiltration (1,041 mm) determined in the field.

**Figure 3.** Soil moisture (%) based on depth.

![Soil moisture (%) vs. Depth](image)

The amount of water initially found in the medium interferes with infiltration processes (PANACHUKI et al., 2006; ALMEIDA et al., 2020), among other factors, such as vegetation cover, soil type and physical properties (OLIVEIRA; SOARES; HOLANDA, 2018). Comparing the current results to soil physical parameters can help better explaining the infiltration behavior observed in the current study.

The Horton model parameters were estimated based on TiR data to generate the adjusted equation capable of describing the behavior of instantaneous infiltration rate according to time, through \( f = 261.01 + (551.65 - 261.01) \cdot e^{-1.41t} \) or \( \dot{f} = 261.01 + (290.64) \cdot e^{-1.41t} \), wherein t is expressed in hours. Based on this model, the final infiltration rate (fc) reached 261.01 mm.h^{-1} and the initial infiltration rate (f0) reached 551.65 mmh^{-1}. According to Brito et al. (1996), these values correspond to soil stable infiltration rate and/or basic infiltration velocity (BIV). According to Bernardo, Soares and Mantovani (2006), this BIV can be classified as very high, since the observed value was higher than 30 mm.h^{-1}.

**Figure 4** shows the curves representing the behavior of real instantaneous infiltration rate (TiR) and those estimated (TiE) by the Horton and Kostiakov models.

Overall, both water TiR and TiE in the soil were high at the beginning of the process, although they decreased overtime. According to Bernardo, Soares and Mantovani (2006), infiltration rate gradually decreases to a constant value, known as BIV, overtime. Although the adjusted Horton equation mathematically indicated BIV of 261.01 mm.h^{-1}, if one analyzes **Figure 4**, it is not possible seeing stabilization in TiR.

TiR results have significantly fluctuated between values higher and lower than the estimated infiltration rates, overtime. The initial TiR was 840 mmh^{-1}, whereas TiE based on the Horton method was 545 mm.h^{-1}. Based on this finding, the model tends to underestimate the initial infiltration–rate value. However, the model tends to overestimate most real instantaneous infiltration rate values, up to 25-minute testing.
Figure 4. Curves of real infiltration rates (TiR) and infiltration rates estimated (TiE) by the Horton and Kostiakov models, based on time.

Although large oscillation was observed in results recorded for rates of real instantaneous water infiltration into the soil and based on the behavior of infiltration curves (Figure 4), it was possible noticing that the Horton model presented satisfactory fit, since the final values tended to stabilize. Furthermore, the representative curve of data estimated by the model was more accurate in showing infiltration behavior in comparison to real data.

Based on the comparison between the behavior of values estimated by the Kostiakov model and those determined in the field, the model tended to underestimate the initial values. After 80-minute testing, it tended to overestimate most of values recorded for real instant infiltration rates.

Similar behavior was observed by Furquim et al. (2020), who concluded that the Kostiakov's model underestimates the initial infiltration rate values, on average, and that it satisfactorily adjusts to real data at the time to estimate BIV. Silva et al. (2019) also got to the conclusion that values estimated by the Kostiakov model, such as AI and instantaneous infiltration rates, have properly represented the infiltration behavior.

Table 2 presents results observed for coefficient of determination, mean squared error, and mean absolute error used to compare the models.

<table>
<thead>
<tr>
<th>Models</th>
<th>Coefficient of determination (R²)</th>
<th>Mean Square Error (MSE)</th>
<th>Mean Absolute Error (MAE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horton</td>
<td>0.80</td>
<td>85.05</td>
<td>56.86</td>
</tr>
<tr>
<td>Kostiakov</td>
<td>0.67</td>
<td>83.30</td>
<td>59.67</td>
</tr>
</tbody>
</table>
Horton's model expressed better coefficient of determination (0.80) and mean absolute error (56.86) results than the Kostiakov model, which recorded coefficient of determination of 0.67 and mean absolute error of 56.86. On the other hand, the Kostiakov model recorded the best result for mean squared error (83.30), whereas the Horton model recorded 85.05 for the same parameter. Paixao et al. (2004) carried out infiltration tests based on using double-ring infiltrometer in Sandy Loam soil. They also observed the best determination coefficient values for the Horton model (0.99), whereas the Kostiakov model recorded 0.95 for this same parameter.

On the other hand, Almeida et al. (2020) used double ring infiltrometer in Cambisol and found coefficient of determination of 0.97 for the Kostiakov model and 0.87, for the Horton model.

Figure 5 presents the heat map of the real infiltration rate estimated in the field and that estimated by the Horton and Kostiakov models based on time.

Based on the heat map, it is possible seeing greater heterogeneity in times (vertically) at the beginning of the test, a fact that explains the difference between models. However, it is possible seeing homogenous times at the end of the test. Neither the Horton nor the Kostiakov model showed difference between real infiltration rates; however, the Kostiakov model was the one that came closer to the real infiltration value.

**Figure 5.** Heat map.
CONCLUSIONS

High water infiltration rate was observed at the beginning of the process, likely due to low moisture level at the topsoil layer.

The Horton model tended to overestimate most real instantaneous infiltration values up to 25-minute testing. The Kostiakov model, in its turn, tended to underestimate the initial infiltration rate values, as well as to overestimate them after 80-minute testing.

Despite the large fluctuation in values observed for real instantaneous infiltration rates, both the Horton and Kostiakov models presented satisfactory fit to the infiltration rate behavior in the experimental area.

Based on the heat map analysis, the Kostiakov model was the closest to real infiltration values.

Further studies on water infiltration in the experimental area should be conducted based on analyses of soil physical properties, to help better understanding the infiltration behavior in this region.

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